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FINAL REPORT

PENETRATION MECHANICS WITH AN ARBITRARY LAGRANGIAN EULERIAN FINITE ELEMENT CODE

by

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and

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U.S. ARMY RESEARCH OFFICE

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Methodologies for Arbitrary Lagrangian-Eulerian (ALE) formulations for the treatment of penetration problems are studied. These methods are attractive because they would permit far more economical simulation of penetration problems by avoiding the extreme crushing of the elements at the penetrator/target interface. Previous to these developments, computer simulations of two and three dimension problems are so time-conumsing (2 to 24 hours on the largest computer) that they cannot be made within the normal framework of engineering analysis and decision making. These methods offer the potential of an order of magnitude reductions in running time. Five aspects of ALE formulations are studied: the convergence and stability of

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multi-time step (subcycling) algorithms, development of ALE constitutive laws which are derivable from Lagrangian constitutive equations, and development of mapping procedures for the mesh so that excessive distortion is avoided. In the research program, these aspects are studied in conjunction with realistic computations of two dimensional penetration problems.

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SUMMARY OF RESEARCH FINDINGS

Research work has concentrated on the development of subcycling methods which will be needed to treat the shrinking elements near the target - penetrator interface and in developing consistent and accurate formulas for updating the stresses for history dependent materials where the material moves through the mesh. A significant breakthrough has been achieved in the first goal in that for the first time a rigorous proof of stability has been developed for a subcycling method for a first order semidicretization. The construction of this proof required an unusual methodology in that the evolution operator had to be bounded on an element basis in the spectral space. This finding, which was begun under an NSF grant, has appeared in the Journal of Computer Methods in Applied Mechanics and Engineering.

One factor which has arisen and still needs to be studied in both the first order and second order systems is to study the basic consistency of these equations with subcycling. The consistency of the time integration is of importance in establishing the convergence of such procedures and whether or not the equations for even the first order semidiscretization are consistent is not readily apparent. However, some recent work undertaken in conjunction with Professor Tom Hughes of Stanford University indicates that work consistency can be proven for these subcycling procedures. This work was accepted for publication in International Journal of Engineering Science.

Tne development of consistent formulations for updating stresses in history dependent materials in meshes that are not coincident with the material has progressed quite well. The approach which has been followed has been to develop weak forms of the governing equations which incorporate the stress rate within an element, so that the updating procedure is variationally consisten and is not made in an ad hoc fashion. This development leads to two journal articles which will appear in the journal of Computer Methods in Applied Mechanics and Engineering.

We have made a substantial breakthrough in our understanding of the behavior of eroding elements by the development of closed form solutions for transient response or rods which have stress-strain curves that are not monotone. The closed form solutions have been compared to finite element solutions and it has been illustrated that the extreme singular behavior associated with non-monotone stress-strain curves leads to vey slow convergence and severe spurious oscillations. These finding will be published in the Journal of Applied Mechanics.

Contact has been maintained on our work on subcycling and erosion with Aberdeen Ballistic Research Laboratory, Dr. Kent Kimsey. We are providing him with the latest information on our research on subcycling and erosion and have incorporated some of these ideas in the EPIC-2 code and they have been tested in penetration problems. Similar plans have been made for erosion algorithms and we have sent these modified codes to Aberdeen Proving Grounds.

Seven (7) journal articles resulting from work under this contract during the period of May 1, 1984 to January 31, 1987 are and/or to be published in professional journals. A list of the publications is included.

Publications

Ted Belytschko, P. Smolinski and W.K. Liu, "Stability of Multi-Time Step Partitioned Integrators for First Order Finite Element Systems, Computer Methods in Applied Mechanics and Engineering, 49, 281-297, 1985.

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T. Belytschko, X.J. Wang, Z.P. Bazant and Y. Hyun, "Transient Solutions for One Dimensional Problems with Strain-Softening," accepted for publication in Journal of Applied Mechanics, ASME.

T.J.R. Hughes, T. Belytschko and W.K. Liu, "Convergence of an Element-Partitioned Subcycling Algorithm for the Semi-Discrete Heat Equation," submitted to International Journal of Engineering Science, in press.

The participating Scientific Personnel

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